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TABLE OF CONTENTS

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AN EVALUATION OF CABLE LACING MATERIAL TO SELECT A SINGLE TYPE FOR USE ON WIRE BUNDLES

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Summary -- A method to assist in the selection of a material for cable lacing bundles is discussed. Heretofore little work has been published evaluating cable lacings, and opinions vary widely as to the proper material to use. This paper is intended to help resolve some of these opinions by presenting the results of an experiment performed at our plant. Five rolls of twine and tape were evaluated to determine which material (base material and coating) would best serve our requirements.

TEST PROCEDURE AND ANALYSIS

In this experiment, five rolls of cord described in Table I were tested for breaking strength and elongation before and after sequential fungi-humidity tests. In addition, two rolls were tested to determine the effect of a knot upon the breaking point and elongation of the cords after undergoing the fungi and humidity tests. A diagram of the knot is shown in Fig. 1. Each roll of cord was cut into 24 inch lengths, and ten strands from each roll were then selected at random for each portion of the tests. These strands were then randomized with respect to each other in order to reduce experimental bias.

TABLE I

Roll	Material	Coating	Type	MIL Designation	Cross-Sectional Area $(in)^2$	Allowed Breaking Strength (MIL-T-713) (pounds)	Allowed Elongation Per Cent (MIL-T-713)
A	Nylon	Wax	Tape	MIL-T-713, tape, type P, waxed, CL 2	0.00096	50 (Avg)	--
B	Nylon	Rubber	Tape	--	0.00099	--	--
C	Nylon	Rubber	Twine	--	0.00049	--	--
D	Nylon	Wax	Twine	MIL-T-713, twine, type P, waxed, CL 2	0.00080	48 (Avg)	20
E	Dacron	Wax	Tape	None	0.00096	--	--

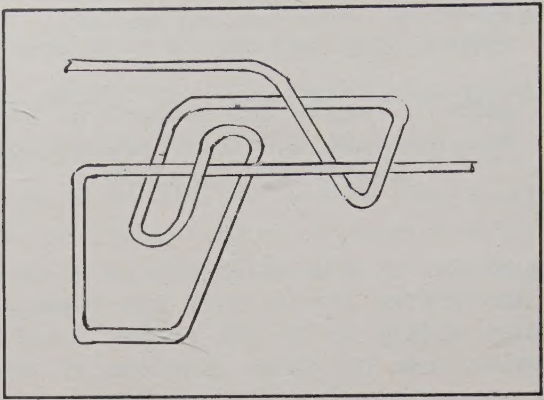

Fig. 1

Upon completion of the fungus test (in which no visible aftereffects were noted) the samples were inspected and then placed in the humidity chamber for a ten day humidity test per MIL-E-5272. Once again, no visible aftereffects were noted. In the compilation of results, a normal distribution was assumed, since the data were insufficient to determine the true distribution and since most naturally occurring distributions are known to approach the normal distribution as the quantity of data (sample size) is increased.

Figures 2 and 3 show the distribution found for each roll, both before and after the fungi-humidity tests. Figure 2 shows the breaking point, while Fig. 3 shows the elongation. If this test were repeated, the distribution shown on these charts would probably vary slightly due to the inherent unrepeatability of most test data. However, it can be predicted that the data will lie within specified limits 90 per cent of the time. These limits, for the mean and standard deviation, are shown in Table II.

Table III shows the mean tensile strength of each roll before and after the fungi-humidity tests. The latter tensile strength is probably inaccurate, since the cross-sectional areas of the cords were not remeasured. The tensile strength in each case is based on the cross-sectional area of the cord as received. Values for roll C are shown to be very high in comparison to the other rolls. Roll C is a rubber covered twine and very smoothly finished. Roll D, which is wax coated twine, is rather unsymmetrical after the coating process. A check of the diameter of the twine before coating (vendor was contacted) revealed that the basic twine stranding is the same in both rolls (cross-sectional area prior to coating equals 0.000122 square inch). It may thus be assumed that the coatings add materially to the thickness and contribute to the very good showing of roll C in Tables III and IV. This information must be relegated to secondary importance in light of other results discussed herein.

Table IV shows the mean modulus at the breaking point for each cord before and after test. This modulus will naturally not be the same as Young's modulus since the elastic limit of each cord has been exceeded. Also shown in Table IV is the ultimate elasticity of each roll. A comparison of data taken before and after the fungi-humidity tests reveals a significant difference (99 per cent confidence or better) between the two with respect to both breaking point and elongation. The fungus and humidity tests affect the cords detrimentally in both cases, except roll A, which is apparently improved by fungi-humidity testing.

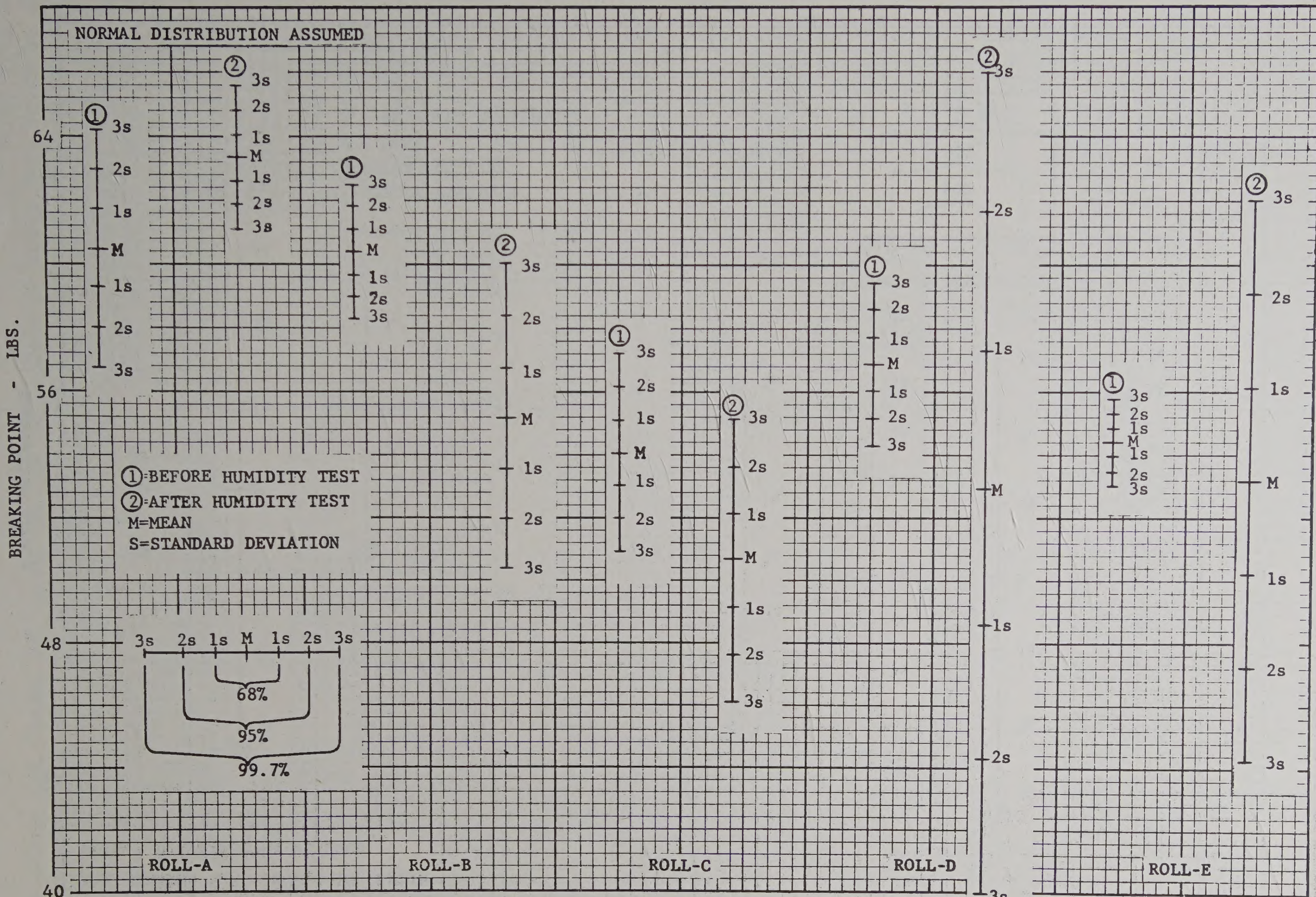

Fig. 2 - Breaking point of twine and tape.

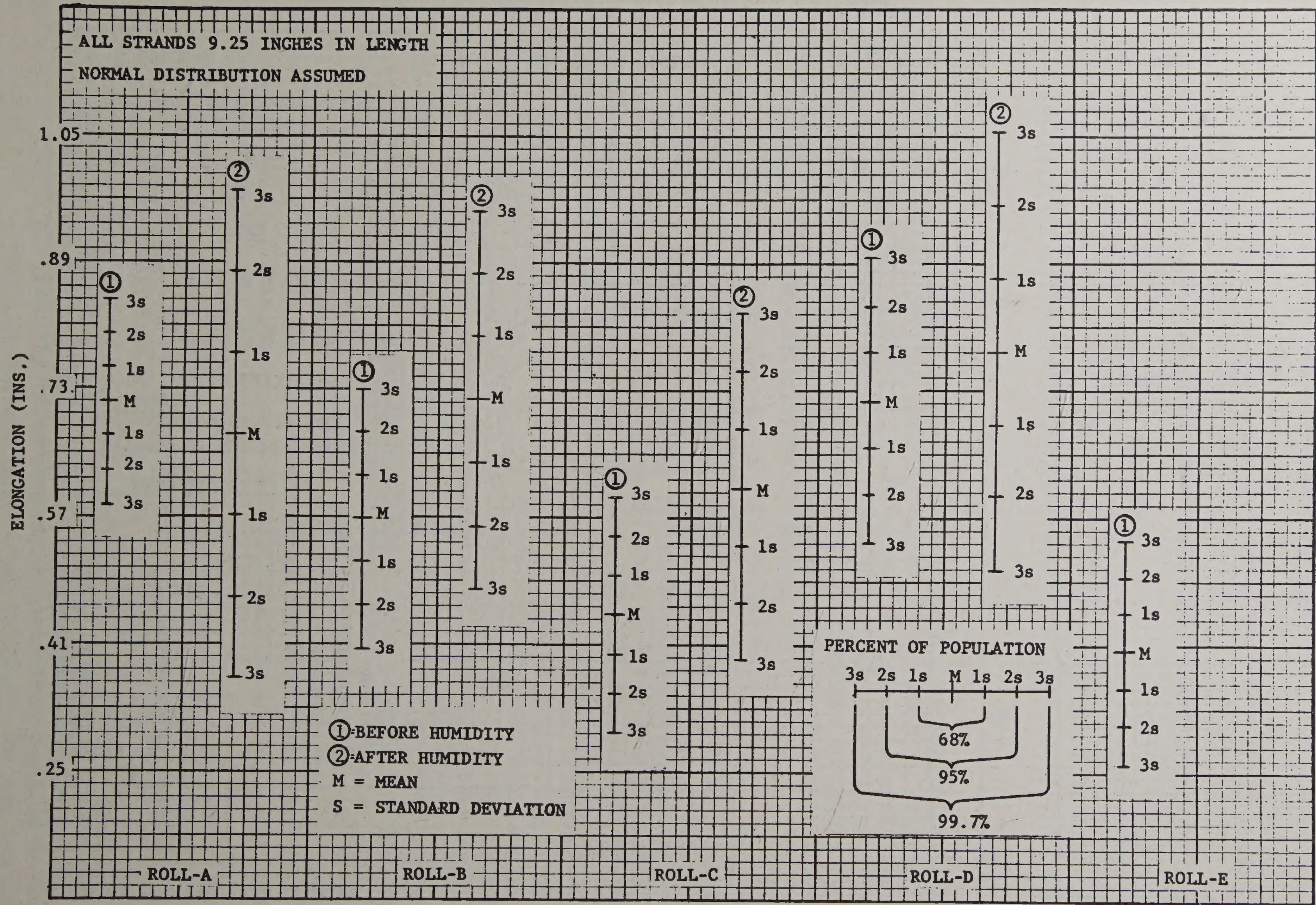

Fig. 3 - Elongation of twine and tape at breaking point.

TABLE II

Minimum and Maximum Parameters (90% Confidence Limit)

Roll	Test	Min. Mean	Max. Mean	Min. Std. Dev.	Max. Std. Dev.
		Breaking Point (Pounds)			
A	Before	59.8	61.2	0.93	2.10
	After	63.0	63.8	0.51	1.15
B	Before	60.0	60.8	0.51	1.15
	After	54.0	56.0	1.24	2.80
C	Before	53.4	54.6	0.77	1.73
	After	49.7	51.5	1.15	2.60
D	Before	56.2	57.2	0.60	1.35
	After	50.4	55.4	3.14	7.09
E	Before	54.0	54.6	0.35	0.79
	After	51.4	54.8	2.19	4.93
		Elongation (Inches)			
A	Before	0.6873	0.7377	0.0316	0.0714
	After	0.6097	0.7279	0.0741	0.1676
B	Before	0.5376	0.6000	0.0387	0.0883
	After	0.6794	0.7706	0.0574	0.1296
C	Before	0.3967	0.4533	0.0360	0.0806
	After	0.5754	0.6622	0.0538	0.1233
D	Before	0.6777	0.7473	0.0435	0.0984
	After	0.7147	0.8229	0.0678	0.1536
E	Before	0.3744	0.4256	0.0319	0.0719
	After	0.4063	0.5155	0.0514	0.1463

The data of the experiment indicate that the knot used on samples from rolls B and D weakens the cord by approximately 42 per cent and reduces elongation by approximately 23 per cent. These results apply only to rolls B and D under the stated conditions. It may be assumed that the other rolls would be similarly affected by knots. A diagram of the test equipment used in the experiment is shown in Fig. 4. Other observations made during the course of the experiment were: (1) Twine is more apt to cut a worker's hands than tape; (2) knots in tape are easier to untie than those in twine; and (3) the fewer the knots, the longer the lacing will last.

TABLE III

Tensile Strength (lb/in^2)

Roll	Before Test	After Test
A	63.02 x $(10)^3$	66.04 x $(10)^3$
B	61.01 x $(10)^3$	55.56 x $(10)^3$
C	110.20 x $(10)^3$	103.27 x $(10)^3$
D	70.88 x $(10)^3$	66.13 x $(10)^3$
E	56.56 x $(10)^3$	55.31 x $(10)^3$

TABLE IV

Roll	Modulus (lb/in^2)		Ultimate Elasticity (%)	
	Before Test	After Test	Before Test	After Test
A	8.18 x $(10)^5$	9.13 x $(10)^5$	7.70	7.23
B	9.92 x $(10)^5$	7.08 x $(10)^5$	6.15	7.84
C	23.98 x $(10)^5$	15.44 x $(10)^5$	4.59	6.69
D	9.20 x $(10)^5$	7.95 x $(10)^5$	7.70	8.31
E	13.08 x $(10)^5$	11.10 x $(10)^5$	4.32	4.98

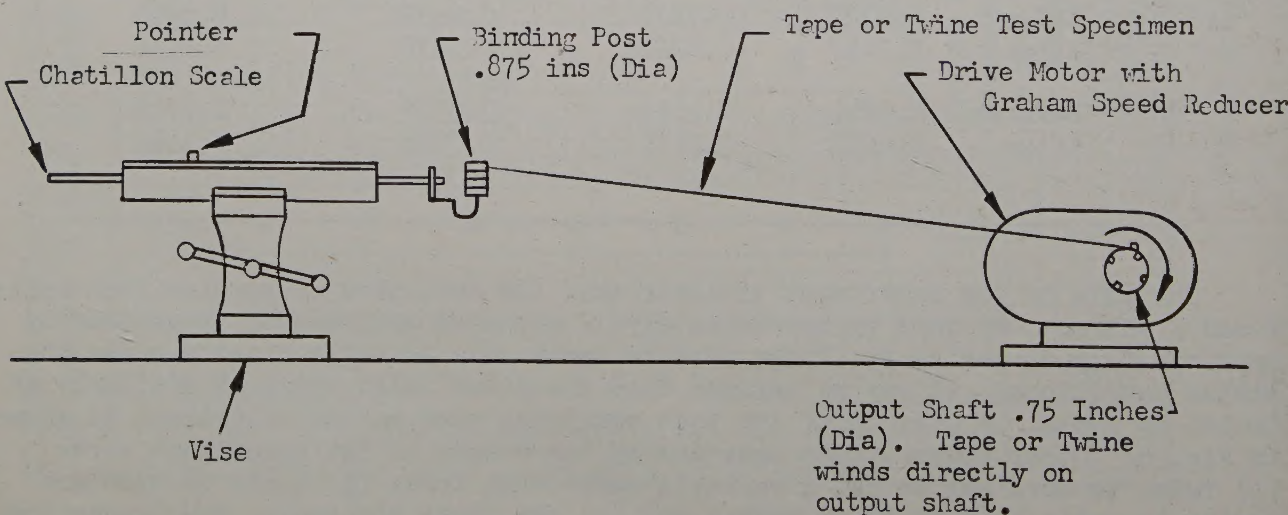

Fig. 4 - Test setup to measure elongation and tensile pull.

CONCLUSION

If MIL-T-713 is used as a basis for comparison, it may be concluded that all specimens are satisfactory, as may be visualized by inserting the specification limits in Figs. 2 and 3. Roll A very definitely proved to be the strongest roll and is the only roll which remains above the MIL-T-713 tensile value before and after tensile testing. The superiority of one type of coating over another cannot be established from this data. Rolls B and C rate about equal, and roll E, which is a wax coated dacron roll, degrades severely in relation to roll A, which is a wax coated nylon roll.

All rolls show significant additional elongation after the humidity-fungus tests. Prior to the fungus-humidity testing, dacron proved to be the superior material, as borne out by Fig. 3 and the ultimate elasticity figure for roll E in Table IV. Based on these findings, wax coated nylon tape is the recommended lacing material. The designation for this material will be found in the "MIL Designation" column of Table I.

APPENDIX

The following statistical methods were used in evaluating the data of the experiment.

Analysis of Variance

A number of different analyses were made in the course of the evaluation. The method used in each case was basically the same. The example given below is by far the most complex analysis performed.

Consider the breaking strength of the cord to be represented by X. Consider also that the effect due to environment, vendor, coating, type of cord, and residual error are represented by subscripts i, j, k, l, and m, respectively. A given breaking strength with respect to the various effects acting upon the cord would, therefore, be represented by X_{ijklm}. If only the effect j were under consideration, the breaking strength values would be represented by $X_{.j...}$, or simply X_j. Other effects upon the breaking strength would be represented in a similar manner.

The variance s^2 of each of these effects or combinations of effects upon the breaking strength of the cord was calculated as follows:

Step 1

$$C = \frac{(\Sigma ijklm\ X_{ijklm})^2}{\eta}$$

where η represents the total number of X values.

$$C_j = \Sigma j\ \frac{(\Sigma jklm\ X_j)^2}{\eta_{jklm}}$$

where η_{jklm} represents the number of X values for each effect j.

$$C_j = \Sigma j \frac{(\Sigma iklm\ X_j)^2}{\eta_{iklm}}$$

where η_{iklm} represents the number of X values for each effect j.

$$C_{ij} = \Sigma ij \frac{(\Sigma klm\ X_{ij})^2}{\eta_{klm}}$$

where η_{klm} represents the number of X values for each group designated by effect i and j.

$$C_{jk} = \Sigma jk \frac{(\Sigma ilm\ X_{jk})^2}{\eta_{ilm}}$$

where η_{ilm} represents the number of X values for each group designated by effect j and k.

$$C_{ijk} = \Sigma ijk \frac{(\Sigma lm\ X_{ijk})^2}{\eta_{lm}}$$

where η_{lm} represents the number of X values for each group designated by effects i, j, and k.

$$C_{jkl} = \Sigma jkl \frac{(\Sigma im\ X_{jkl})^2}{\eta_{im}}$$

where η_{im} represents the number of X values for each group designated by effects j, k, and l.

$$C_{ijkl} = \Sigma ijkl \frac{(\Sigma m\ X_{ijkl})^2}{\eta_{m}}$$

where η_{m} represents the number of replicate measurements taken under the combined test conditions.

$$C_{ijklm} = \Sigma ijklm\ (X_{ijklm})^2.$$

<u>Step 2</u>

$$s_i^2 = \frac{C_i - C}{p - 1}$$

where p represents the number of different environments.

$$s_j^2 = \frac{C_j - C}{q - 1}$$

where q represents the number of different vendors.

$$s_{ij}^2 = \frac{C_{ij} - C_i - C_j + C}{(p-1)\ (q-1)}$$

$$s^2_{k(j)} = \frac{C_{jk} - C_j}{\sum_\alpha q\,(r_\alpha - 1)}$$

where r_α represents the number of measurements in each group designated by effect k.

$$s^2_{ik\,(j)} = \frac{C_{ijk} - C_{ij} - C_{jk} + C_j}{\sum_\alpha q\,(r_\alpha - 1)\,(p-1)}$$

$$s^2_{l\,(jk)} = \frac{C_{jkl} - C_{jk}}{\sum_{\alpha\beta} q r_\alpha\,(v_\beta - 1)}$$

where v_β represents the number of measurements in each group designated by effect l.

$$s^2_{il\,(jk)} = \frac{C_{ijkl} - C_{ijk} - C_{jkl} - C_{jk}}{\sum_{\alpha\beta} q r_\alpha\,(v_\beta - 1)\,(p-1)}$$

$$s^2_{m\,(ijkl)} = \frac{C_{ijklm} - C_{ijkl}}{\eta + 1 - \sum_{\alpha\beta} q p r_\alpha\, v_\beta - p - q}$$

The variances s^2_i, s^2_j, $s^2_{k(j)}$, $s^2_{i(j)}$, $s^2_{ik(j)}$, and $s^2_{il(kj)}$ were compared to the variance $s^2_{m(ijkl)}$ in order to obtain Snedecor's F value.[1] The variance $s^2_{m(ijkl)}$ was chosen as the denominator on the basis of a method used by C. R. Hicks.[2] The various ratios (F values) were then compared to Merrington and Thompson's table[3] on the F distribution for significance.

Comparison and Classification of Means

Means were compared by a method developed by Tukey.[4]

Mean and Standard Deviation

The standard deviations were obtained as follows:

$$s = \sqrt{\frac{\Sigma x^2 - (\Sigma x)^2/\eta}{\eta - 1}}$$

where s = standard deviation
x = measured value
η = number of measured values.

The minimum and maximum means were obtained by

$$M - \frac{ts}{\sqrt{\eta}} \leq M \leq M + \frac{ts}{\sqrt{\eta}}$$

where M = mean

t = a value obtained from Merrington and Thompson's table[3] using a significance level of 0.05.

The minimum and maximum standard deviations were obtained by use of the following equations:

$$s \text{ max.} = s\sqrt{\frac{\eta - 1}{x^2 (\eta-1),\ 0.975}}$$

$$s \text{ min.} = s\sqrt{\frac{\eta - 1}{x^2 (\eta-1),\ 0.025}}$$

where x^2 is a value obtained from Thompson's table of x^2 values.[6]

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THE FITTING OF A FAILURE-RESPONSE CURVE TO EXPERIMENTAL DATA

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In radiation experiments, effects among components are measured during and/or after irradiation with known dosages. These components form a sample from the population of components. The fitting of the dose-failure curve is an attempt to infer from a given experiment the conditions obtained in a class of components. The calculated regression line of the dose-failure diagram is the most accurate estimate that can be drawn from the data, granted that the basic assumptions are correct.

To measure susceptibility to radiation, the functional threshold, or smallest effective dose, is established. This threshold will vary from sample to sample. The dose-failure curve is usually sigmoid in shape; however, a transformation may be necessary to change the curve to the sigmoid shape. A further transformation will establish a linear relation between log of dose and log of the ratio of nonfailures to failures, as shown below in Eq. (3).

If a sample of components of a particular type are irradiated at a constant rate and the times recorded at which the individual components cease to function properly, the transformed dose-failure curve can be drawn. From these data and the regression curve, the functional threshold can be estimated. In a "one-shot" experiment, the total sample should be divided into groups, or subsamples. Then a separate group of ten or more should be tested at each dose level. The percentage of failures at each dose level will establish the regression line.

Although the log transform is not an essential part of the treatment, it is empirically known to be desirable in many cases. Each new case requires rejustification. The final transformed linear relation can be considered as a simple case: $y = ax + b$, where x is log dose. If neither x nor y were subject to any uncontrolled, or error, variation, the problem would be easy. But the estimate depends on the types of error to which each variable is subject. The variable x is subject to errors of dosimetry and y suffers from the many errors which are elsewhere considered.

The precision of the results depends on the variation of the effects obtained. To estimate the errors, one must obtain independent estimates of the dose used in the experiment and estimate the errors in the parameters involved. The response of an individual component may be a continuous variate, or it may be an all-or-none relationship. The response of a group of components may be the average response or the per cent of components giving the characteristic response. The variation in response need not be the same at all levels.

The characteristics of the dose-response relationship are:

1. Very low doses elicit no response.
2. Very high doses elicit total response.

3. Between these two doses there exists a point of maximum rate of change of response.

4. The middle range of the dose-response curve is the steepest part of the curve.

DISCUSSION

If a group of N components are subjected to radiation at a constant rate and the failure time for each component is recorded, the per cent of the total N that fail in a given time is a measure of the radiation effect. The statistical distribution of failures in terms of the amount of radiation can then be estimated.

Assume there exists a critical amount of radiation associated with each individual component, at which that component fails. Record the time at which each component fails while being irradiated at a constant rate of irradiation. The components that have failed at any time have a tolerance lower than the integrated amount of radiation measured at that time. The parameters of the distribution of tolerances can be estimated by comparing the per cent failing at each failure time.

If the total number of failures during the experiment is f, then the total fraction failing is f/N, or 100 f/N per cent. If the distribution of failures is plotted against the total amount of radiation, the ordinates will increase in uniform steps of 1/N, but the abscissas will fall at irregular points.

Let x be the total amount of radiation and y the fraction of failures. Then, there is a relation between x and y which can be expressed, $y = F(x)$. Suppose for simplicity that the function is linear; i.e., $y = F(bx + a)$. If the nature of the function can be determined, then the remaining problem is to estimate b and a, the parameters of the distribution.

The principle of linear regression can be applied to this problem. This principle describes the change in y due to arbitrary changes in x. If the plot of the failure data is sigmoid in shape, then a transformation is necessary to convert the curve into a straight line and simplify the estimation of the parameters of the function. Various methods can be used to determine a transformation to accomplish the above purpose. It may be necessary to let $u = \log_{10} x$, and then determine $y = F(bu + a)$.

If y is the fraction of failures at any particular time, then $(1 - y)$ is the fraction that has not failed. Define the frequency function $f(bu + a)$ as the derivative of $F(bu + a)$; i.e.,

$$f(bu + a) = F'(bu + a) \tag{1}$$

then

$$y = \int_{-\infty}^{u} f(bu + a)\, du \,. \tag{2}$$

If $y' = -by(1 - y)$ where b is a constant, then the solution of the above differential equation is:

$$\ln (1 - y) - \ln y = bu + a$$

or

$$\ln \frac{1 - y}{y} = bu + a \ , \qquad (3)$$

where a is the arbitrary constant. And if

$$z = \ln \frac{1 - y}{y}$$

then $z = bu + a$ is the linear regression function required.

Solving this equation for y gives

$$y = \frac{1}{1 + e^{bu + a}} \qquad (4)$$

This function is called the "logistics function." It is similar in shape to the Gaussian distribution, but simpler in some respects to apply.

Since the effect of amount of radiation is frequently determined on a logarithmic scale, it may be necessary to make several transformations. To illustrate: if x is the integrated flux in neutrons/cm^2, let $t = \log_{10} x$. A further transformation would be to normalize the variables by

$$u = \frac{t - \bar{t}}{k} \qquad (5)$$

where $\bar{t}$ is the mean of the t's and k is the unit of measure desired.

To estimate the parameters a and b, the method of least squares can be used. The quantity $\bar{z}$ would be an estimate of a. Since (1 - y) and y are equal at the mean, the value of $\bar{z}$ is zero in this case. Hence, only the parameter b need be estimated. By the method of least squares, an estimate of b is

$$\frac{\sum u_i \, z_i}{\sum u_i^2} \ . \qquad (6)$$

Another method of estimating the parameter b, which can be done much more rapidly, but less accurately, will also be illustrated. The steps in this rapid method are:

1. Justify the use of the logistics function.
2. Plot the radiation-failure response data.
3. Select one point near each end of the failure data to use in estimating the parameter b.
4. Solve for the parameter b, and check solution by comparing with additional points on the curve.

Each of these two methods will be illustrated by an example. The longer, more deliberate method should be used where greater precision is desired.

Method 1

To illustrate the rapid method of estimation, data from OHNL 1720, December 6, 1955, are used. These data show the effect of radiation on primers firing

by stab action. Figure 1 shows x, the height of fall of a 3.95 ounce ball measured in inches as the independent variable, and y, the per cent of primers firing.

The x values can be simplified by the transformation:

$$u = \frac{x - x_o}{.25}$$

where x is in inches and u is the number of increments on the abscissa scale.

The experimental data points for unirradiated primers are tabulated in Table I. An arbitrary value of x_o has been selected. The value of u which will give 50 per cent response is chosen and designated m. By interpolation

$$m = \frac{1 + 2}{2} = 1\tfrac{1}{2} .$$

The fitted curve will be symmetric about the point m.

TABLE I

Unirradiated Primers

x	u	y
0.50	-1	1%
0.75	0	12%
1.00	+1	20%
1.25	+2	80%
1.50	+3	100%

To fit a logistics curve to the data in Table I, note that from Eq. (4)

$$y = \frac{1}{1 + e^{bu + a}}$$

$$\frac{1}{y} = 1 + e^{bu + a}$$

$$\frac{1 - y}{y} = e^{bu + a}$$

Let a = -bm. Then, repeating Eq. (3),

$$\frac{1 - y}{y} = e^{b(u - m)} .$$

The value m = 1.5 can be checked at

$$u = 1.5$$

$$e^{b(1.5 - 1.5)} = e^{o} = 1 = \frac{1 - y}{y}$$

or

$$2y = 1$$

$$y = .50 = 50\%,$$

which satisfies the condition. To estimate b, choose the point u = 1

$$y = 20\% = 1/5 .$$

Then, substituting the value of y in Eq. (3)

$$\frac{1 - 1/5}{1/5} = 4 = e^{b(1 - 3/2)}$$

$$4 = e^{-1/2\ b}$$

$$-1/2b = 1.39$$

$$b = -2.78$$

check by using

$$u = 2 \qquad 6 = 80\% = 4/5$$

then

$$\frac{1 - 4/5}{4/5} = 1/4 = e^{-2.78(2 - 3/2)} = e^{-1.39} = 1/4 .$$

The fitted curve

$$y = \frac{1}{1 + e^{-2.78(u - 3/2)}}$$

is plotted in Fig. 1. It fits the data closely except at the point x = .75. To illustrate a method of obtaining a closer fit, two curves are fitted to the data shown in Table II. These data show the effect of irradiation on this type of primer.

TABLE II

Primers After Irradiation

x	u	y	Fitted y
0.50	-1	0.05	0.05
0.75	0	0.52	0.50
1.00	+1	0.77	0.77
1.25	+2	0.97	0.945
1.50	+3	1.00	0.973

Here m is approximately equal to zero. Hence

$$\frac{1 - .05}{.05} = e^{b(-1)}$$

$$19 = e^{-b}$$

$$b = -2.94$$

$$y = \frac{1}{1 + e^{-2.94\,u}} \quad \text{for } u \leq 0 \,.$$

Therefore, if

$$u = 1$$

$$y = .77 \,,$$

then

$$\frac{1 - .77}{.77} = .3 = e^{b} = e^{-1.2}$$

$$b = -1.2$$

$$y = \frac{1}{1 + e^{-1.2\,u}} \quad \text{for } u \geq 0 \,.$$

A very close fit is achieved by defining the response curve in two sections, as shown in Fig. 1.

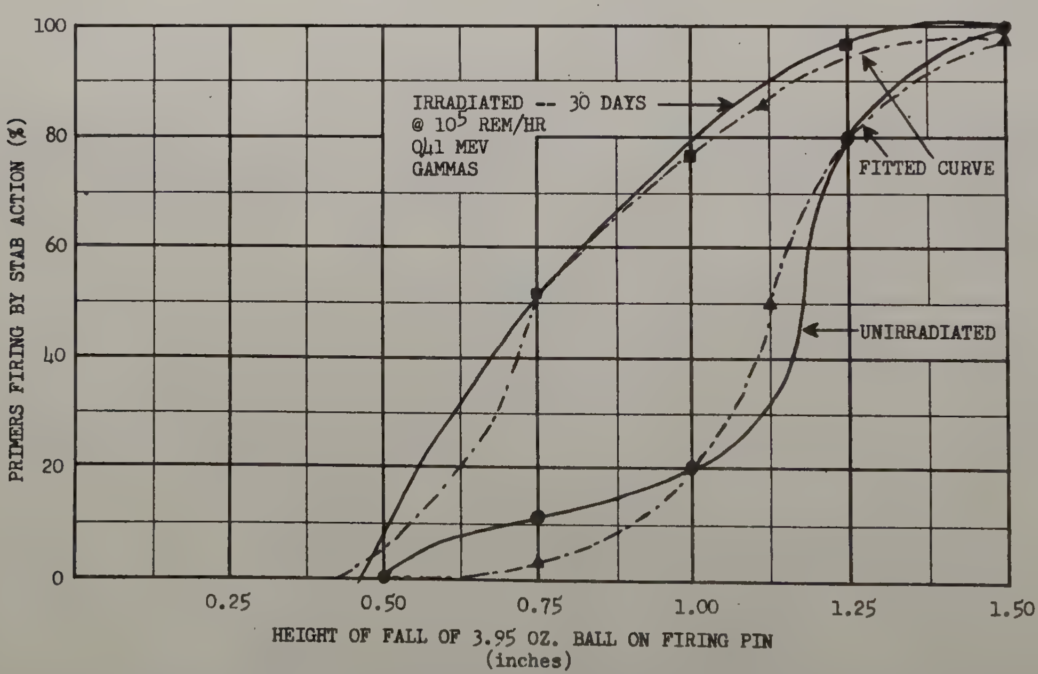

Fig. 1 - Sensitivity change of irradiated M26 primers at ambient temperatures.

Method 2

Data to illustrate the least squares method of estimating the parameter b are from Admiral tests of radiation effects on semiconductors. Twenty-five transistors were tested to determine the fast neutron integrated flux at which each transistor failed. There were 22 failures at an integrated fast neutron flux less than 2.7×10^{13} neutrons/cm^2. Let x be the integrated neutron flux in neutrons/cm^2. Let $t = \log_{10} x$ and $\bar{t}$ = average value of t. The transformations $u = t - \bar{t}$ and

$$z = \ln \frac{1 - y}{y}$$

lead to the functional relations that are required. The values of x, t, u, $(1 - y)/y$, and z are shown in Table III. Applying the method to estimate b gives

$$y = \frac{1}{1 + e^{-2.1\,u}} \;.$$

This curve is plotted in Fig. 2 and the failure points are also shown.

By plotting the theoretical or fitted distribution of failures curve, it is possible to predict the tolerance functional threshold of the group of transistors. The functional threshold value below which no transistor would be expected to fail is read from Fig. 2:

$$x_t = 9 \times 10^{10} \text{ neutrons/cm}^2 \;.$$

Further simplifications of the application of Method 2 are possible by plotting a family of curves on transparent log paper and using these curves as templates for estimating the equation to fit the experimental data.

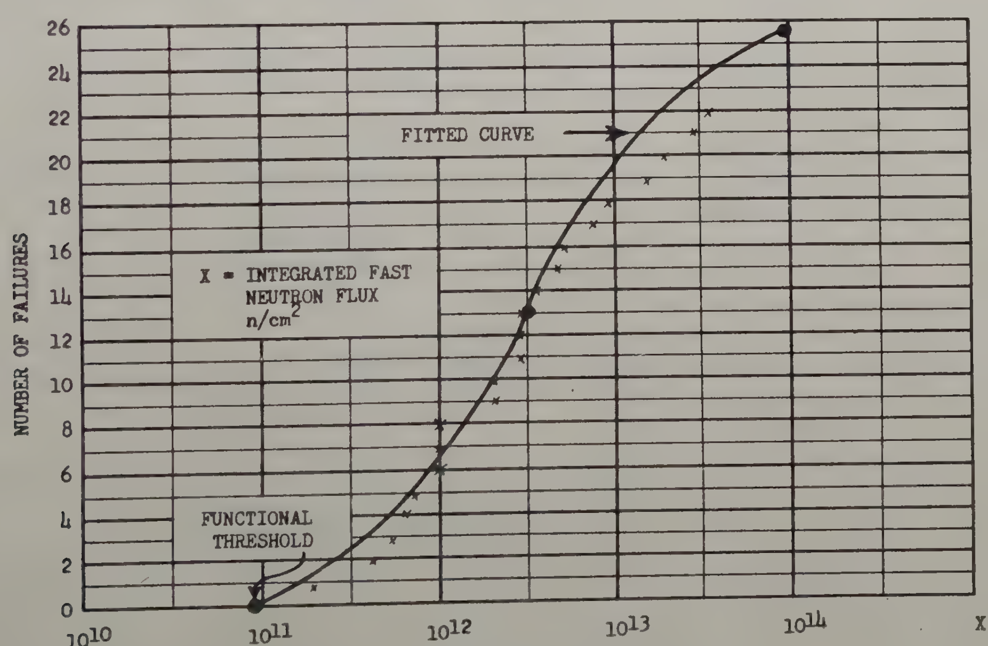

Fig. 2 - Transistor failures.

TABLE III

Transistor Failures from 25 Transistors Tested

Failure Number	x = Flux n/cm^2	$t = \log_{10} x$	$u = t - \bar{t}$	$\frac{1-y}{y}$	$Z = \ln \frac{1-y}{y}$
1	2.0(11)*	11.301	-1.115	24	3.178
2	4.0	0.602	-0.814	23/2	2.442
3	5.5	0.740	-0.676	22/3	1.992
4	6.4	0.806	-0.610	21/4	1.658
5	7.5	0.875	-0.541	20/5	1.386
6	1.0(12)	12.000	-0.416	19/6	1.153
7	1.0	0.000	-0.416	18/7	0.940
8	1.0	0.000	-0.416	17/8	0.753
9	1.2	0.079	-0.337	16/9	0.571
10	1.2	0.079	-0.337	15/10	0.405
11	2.7	0.431	+0.014	14/11	0.239
12	2.9	0.462	+0.045	13/12	+0.077
13	3.0	0.477	+0.060	12/13	-0.077
14	3.6	0.556	+0.139	11/14	-0.239
15	4.5	0.653	+0.236	10/15	-0.405
16	5.0	0.698	+0.282	9/16	-0.571
17	8.0	0.903	+0.486	8/17	-0.753
18	9.5	0.977	+0.560	7/18	-0.940
19	1.6(13)	13.304	+0.787	6/19	-1.153
20	1.9	0.278	+0.861	6/20	-1.386
21	3.0	0.477	+1.060	4/21	-1.658
22	3.7	0.568	+1.151	3/22	-1.992

*Denotes 2×10^{11}.

$\Sigma = 273.17141$

$\bar{t} = 12.41688$

$\bar{x} = 2.611 \times 10^{12}$ n/cm^2

$\Sigma u_i z_i = -17.5$

$\Sigma u_i^2 = 8.31$

$$b = \frac{\Sigma u_i z_i}{\Sigma u_i^2} = \frac{-17.5}{8.31} = -2.1$$

Hence

$$y = \frac{1}{1 + e^{-2.1\,u}}$$

APPLICATION OF A METHOD OF INSPECTION TESTING TO ASSURANCE RELIABILITY

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Summary -- This paper describes the use of a sampling method of inspection testing for the purpose of controlling reliability in a system. The acceptable quality level (AQL) and mean failure free time can be estimated by the maximum likelihood method, assuming that errors are distributed by the Poisson Law. Hence, a choice of quality levels or reliability levels can be made. Single or multiple sampling plans may be used depending on the cost factors, the sample size, and the available information on reliability. The sample size, acceptance number, and cost due to Type I and Type II errors are each considered. One aim of inspection testing is to determine when preventable failures occur.
An inspection lot in this case is a test run of a system for a given time. The system is accepted or rejected on the basis of the number of failures found in the sample tests.

The object of the inspection system is (1) to accept the reliability of the system at least 100 (1-A) per cent of the time when the true life is more than T_1 and (2) to accept the reliability of the system not more than a small proportion, B, of the time when the true mean life is less than T_2, $T_2 < T_1$.

DEFINITIONS

AQL	=	Acceptable quality level.
T	=	Failure-free time, mean life, expected time to failure.
$\hat{T}$	=	Estimate of the mean life.
C	=	Acceptance number. A value of time to failure which, if exceeded by the sample mean failure free time, will assure the required reliability coefficient.
r	=	Number of failures.
Inspection	=	Measuring, examining, testing, gaging, or otherwise comparing the reliability of the system with the required standard.
Type I Error	=	Vendor's risk. The probability of rejecting the reliability of the system, even though the reliability is satisfactory.
Type II Error	=	Convair's risk. The probability of accepting the reliability of the system, when the actual reliability is not as good as the specification.

INTRODUCTION

A weapons system is composed of one or more subsystems. Each subsystem is subject to a reliability specification by a vendor and acceptance by Convair. It is certain that there will be failures in the system, though the failure rate is not known. The user is faced with the problem of setting up standards for accepting the vendor's subsystem. He wishes to be assured with a degree of confidence (1-A) that the reliability of the subsystem will be at least as good as R.

Theoretically, this is the general problem of statistical decision, as formulated by Wald. The true reliability of the subsystem cannot be determined. But there is a random variable T (mean time to failure) whose distribution depends on the true situation. An observed value of T provides information about the true condition of the subsystem, and it can be used to increase our chances of making a good decision. The general problem is that of choosing a decision function $\delta(T)$, relating the sample space to the decision space.

Adequate control can be attained by the application of a program of inspection and testing. Now, the variety of plans, procedures, and tables for a program is unlimited. The conditions affecting a program need to be carefully stated, and a plan must be determined which minimizes the maximum risk. This is the Minimax principle. This paper presents a plan for testing a hypothesis regarding the reliability of a system. A sequential testing program might also be devised to determine a system's reliability.

Before a sampling procedure is set up, several questions should be answered:

1. How long should each test be?
2. How many subsystems are required to be tested?
3. What variables will be modified?
4. What type of sampling should be employed, single or multiple?
5. What is the risk function (the expected loss to the vendor and the user) if the Type I error is a function of weight, cost, performance, etc., and the Type II error is the probability of catastrophic failure?

The procedure should determine the degree of the reliability of the system or subsystem. The specified reliability of the subsystem should be made binding, and provisions should be set forth to be followed to measure such reliability.

STATEMENT OF THE PROBLEM

The purpose of this paper is to present a rational scheme for determining the reliability of a subsystem as a basis for the decision whether to accept that reliability or not. We define inspection as the process of measuring, examining, testing, gaging, or otherwise comparing the reliability of the subsystem with the standard required. A failure is any deviation of the functioning of a part or subsystem from the requirements of the specifications, drawings, purchase descriptions, contract, or order. A critical failure is one that could prevent the performance of the tactical function of the system.

Assume that the subsystem, before a test run, is checked and serviced, that the probability of failure is small, and that failures are distributed by the Poisson Law. And consider a random variable, T, the mean time to failure. A test run is a trial with the possible outcomes of "failure" or "no failure." We can think of this as a Poisson trial since the distribution of failures is $P(x) = e^{-u} u^x/x!$ where $u = t/T$ and T = the mean time to failure. The probability of no failures in time t is $P(0) = e^{-t/T}$ since $0! = 1$. Hence, the probability of one or more failures is $Q(0) = 1 - P(0) = 1 - e^{-t/T}$, and $dQ(0) = 1/T\ e^{-t/T}\ dt$ is the probability of failure in time dt, or the rate of occurence of failures. The distribution of a sequence of Poisson trials, given by the failure distribution P(x), is thus approximated by the continuous exponential function $dQ(t)/dt$ or $f(t) = 1/T\ e^{-t/T}$.

To estimate the mean time to failure T, a sampling scheme is devised. The maximum likelihood and minimum chi-square methods of estimation both lead to $E(x) = \bar{x}$ as the estimate of the Poisson parameter u, with minimum variance. We might also consider u as a variable of Pearson type III. However, we know that $\bar{x}$ has a Poisson distribution if the parent variable x is distributed by the Poisson Law. In the continuous exponential distribution, we wish to estimate the parameter T.

Since we are approximating the Poisson distribution by the given exponential function f(t), we investigate the expectation of t:

$$E(t) = \int_0^\infty t\ f(t)dt = \int_0^\infty t/T\ e^{-t/T}\ dt = T, \text{ the mean life.}$$

The higher moments can be obtained by using the Laplace transform, since $L(t^k) = \Gamma(k + 1)\ T^k$. To verify this equation, let $t/T = x$, then $dt = T\ dx$. Then $f(t)\ dt \longrightarrow e^{-x}\ dx$ and

$$\int_0^\infty t^k/T\ e^{-t/T}\ dt \longrightarrow \int_0^\infty T^k\ x^k\ e^{-x}\ dx = T^k\ \Gamma(k + 1)$$

since

$$\Gamma(k - 1) = \int_0^\infty x^k\ e^{-x}\ dx.$$

The problem then is: given a function $f(t,T) = 1/T\ e^{-t/T}$, $t > 0$, $T > 0$, determine a rule of action to (1) accept the reliability of the system with probability $1 - A$ if $T \geqslant T_1$ and (2) reject the reliability of the system with probability $1 - B$ if $T \leqslant T_2$ where $T_2 < T_1$. Since we know the moments and the distribution of T, we can design this decision function.

ANALYSIS

In formulating a plan for reliability assurance, the sampling procedure must be decided, and the risks of either taking unnecessary action or of failing to take necessary action for a given shift in reliability must be determined. The acceptance number for a fixed sample size must also be established.

A single sampling plan has two parameters, sample size and acceptance number. We first decide how many systems will be tested and how long each test will be; this is the sample size. The failure rate can be determined as a grand mean from the total running time and the total number of failures. A reliability chart could be designed to show the expected failure rate for "reliable" operation. The primary consideration is the measurement of time between failures. The mean time between failures can be easily determined from the times between individual failures. We merely divide the total operation hours by the total number of failures.

The sampling scheme can assume a sequential nature, as data are accumulated. However, where in classical theory sample size is determined in advance, in sequential analysis the sample values determine the size of the sample needed to reach a decision. If a trial run continues to failure, the run length L is arbitrary. We can determine the reliability $R = e^{-L/T}$, the probability of no error in a run of length L. The probability of one or more errors is then $1 - e^{-L/T}$.

When making a statistical test of a hypothesis, there are two types of error. In a Type I error, the test run may have a number of failures greater than the limit adopted, although the reliability is satisfactory. The higher the limit adopted, the smaller is this probability. Type II error arises when the reliability of the system is not adequate, but the number of failures is less than the limit set up and no action is taken. This error depends on the shift in the value of the parameter.

Normal inspection is continued as long as the system is functioning in the neighborhood of AQL (the average number of failures we expect). If the reliability is consistently better than AQL, the testing program may be shortened. However, if failures are numerous, an extended testing period may be necessary to give protection against accepting an unreliable product.

We assume the system is from an infinite population of homogeneous products whose failure rate is equal to the process average failure rate which is constant during the inspection period. We are willing to accept the system when the true failure rate is actually not different from the average number of failures. When the failure rate is different, a new test program may be necessary. The average failure rate is used as the AQL. We are interested in the mean failure free time, the reciprocal of the average failure rate, which is estimated from the sample.

SAMPLE SIZE, ERRORS, AND RISK

One problem that arises is that of determining the most economical number of systems to use in the sample if we are given the probabilities of accepting the system, if reliable, and of rejecting it, if unreliable. We can estimate the cost of additional testing due to rejecting the reliability of the system. This, multiplied by the probability of such an error, gives the expected cost of a Type I error. The cost of accepting an unreliable system multiplied by the probability of such an error for a given reliability level gives the expected cost of the Type II error. The sum of these two expected costs gives the total cost for the given test program and reliability limits. We would like to find optimum limits such that the greatest risk is minimized.

Suppose we wish the reliability coefficient R to be 0.70. If R is as good as 0.70, we should like 95 per cent of the time to reach a decision to accept the reliability of the system as satisfactory. The Type I error is 0.05, the probability of rejecting the product even though it is reliable. If R is less than 0.70, the reliability is not adequate and 90 per cent of the time we wish to reach the decision to reject the product as unsatisfactory. The Type II error is 0.10, the probability of accepting a system which is not as reliable as we would like it to be.

We can then set up the risk function: risk = 5 per cent of the cost of continuing testing plus 10 per cent of the cost of accepting a system which is not reliable, plus the cost of the test program. It may be necessary to average the expected cost for all possible shifts in reliability level, weighing the average according to the frequency of the various shifts. The sum of the two expectations gives the total cost due to errors for given sample size and control limits. Now we would like to minimize the cost.

PROCEDURE FOR DETERMINING RELIABILITY

To determine the reliability of a given system, we want to test the hypothesis H_0: $T \geqslant T_1$ against the alternative H_1: $T = T_2 < T_1$. Our rule of action is to accept H_0 if $\hat{T} > C$ and reject H_0 if $\hat{T} < C$. We want 1-A per cent of the time to accept H_0 if true and $\leqslant$ B per cent of the time if H_1 is true. A and B are the errors of the first and second kind. It is possible to find values of r and C if we are given the ratio T_1/T_2 and A and B. The information necessary for making these decisions is given in Table I.

In many cases we can neglect the first r-1 failures and just consider the time of the r^{th} failure. Therefore, it is possible to set up a sequential sampling procedure to decide on the length of time that testing must be continued to obtain the information necessary to reach a decision regarding the reliability of the system. Since $1/T_2 - 1/T_1 > 0$, the region of rejection for $T = T_1$ is $\hat{T} < C$. So we choose C such that $Pr\ (\hat{T} < C/T = T_1) = A$.

We consider the N component parts of the system as a sample of N items drawn at random from some infinite population. We test until some component fails. The component is replaced by a new component. This is sampling with replacement. Each failure will determine the time of the end of a test run. The tests to failure can then be ordered, and the first r times to failure can be designated as $t_1, t_2 \ldots t_r$.

Epstein and Sobel[1] have shown that the maximum likelihood estimate of T in the distribution $f(t) = 1/T\ e^{-t/T}$ is

$$\hat{T} = \sum_{i=1}^{r} \frac{t_i + (n-r)t_r}{r}.$$

This estimate is unbiased and has a chi-square distribution which depends only on r. The probability density function of $\hat{T}$ is

$$f(\hat{T}) = \frac{1}{(r-1)} \left(\frac{r}{T}\right)^r \ (\hat{T})^{r-1}\ e^{-r\hat{T}/T}$$

TABLE I

T_1/T_2	r	A	B	C/T_1
3/2	67	0.05	0.05	0.808
	55	0.05	0.10	0.789
	52	0.10	0.05	0.827
	41	0.10	0.10	0.806
2	23	0.05	0.05	0.6834
	19	0.05	0.10	0.655
	18	0.10	0.05	0.712
	15	0.10	0.10	0.6866
5/2	14	0.05	0.05	0.6046
	11	0.05	0.10	0.5608
	11	0.10	0.05	0.6383
	9	0.10	0.10	0.6036
3	10	0.05	0.05	0.5426
	8	0.05	0.10	0.4976
	8	0.10	0.05	0.582
	6	0.10	0.10	0.5253
4	7	0.05	0.05	0.4694
	6	0.05	0.10	0.4355
	5	0.10	0.05	0.4865
	4	0.10	0.10	0.4363
5	5	0.05	0.05	0.394
	4	0.05	0.10	0.3416
	4	0.10	0.05	0.4363
	3	0.10	0.10	0.3673

from which we determine limits for the estimates of T. We wish to choose a constant C such that if $\hat{T} > C$ we can state that the reliability of the system will meet the specification 1-A per cent of the time.

If we made the transformation

$$\hat{T} = \frac{Ty}{2r}, \quad d\hat{T} = \frac{T}{2r}\,dy$$

then $f(\hat{T})$ becomes

$$g(y) = \frac{1}{(r-1)!}\left(\frac{r}{T}\right)^r \left(\frac{Ty}{2r}\right)^{r-1} e^{\frac{-rTy}{2rT}} \frac{T}{2r} = \frac{1}{2\,\Gamma(r)}\left(\frac{y}{2}\right)^{r-1} e^{-y/2}.$$

This is the well-known chi-square distribution with 2r degrees of freedom. To determine the value of the variable such that $P\ (T < C/T = T_1) = A$, we wish

$$P\left[y \geqslant \frac{2rC}{T_1}\right] = 1-A.$$

From the table of the chi-square distribution, we observe the column for the 1-A level and for 2r degrees of freedom. Set this value equal to $2rC/T_1$ and determine

$$C = \frac{T_1 \chi^2_{1-A}}{2r}.$$

To establish the Type II error, the probability of accepting a system which is not reliable when it passes the required test, we observe the probability that $\hat{T} > C/T = T_2$. We wish this error to be less than or equal to B. If

$$\hat{T} > C = \frac{T_1 \chi^2_{1-A}}{2r}$$

then we want to know for what value of T_2 is the probability that

$$\hat{T} > \frac{T_1 \chi^2_{1-A}}{2r} \leq B$$

true, given that $T = T_2$.

Consider $P\ (Y > 2rC/T_2) = B$. From the table of the chi-square distribution for the B level and 2r degrees of freedom, it is found that $P\ (Y > 2rC/T_2) = B$ holds if

$$\frac{2rC}{T_2} = \chi^2_B \text{ or } C = \frac{T_2 \chi^2_B}{2r}.$$

Then if $P\ (Y > 2rC/T) \leq B$, it follows that

$$\frac{T_1 \chi^2_{1-A}}{2r} \geq \frac{T_2 \chi^2_B}{2r} \text{ and } \frac{T_2}{T_1} \leq \frac{\chi^2_{1-A}}{\chi^2_B}$$

for 2r degrees of freedom. See Fig. 1.

The right-hand side of this expression is monotonic, increasing as r increases. We know the left-hand side of the equation is less than unity. Since the ratio of chi-squares approaches unity from below, there will be a smallest r such that the inequality is satisfied. This will then be the required test as outlined above. Table I shows an example of various ratios of T_1/T_2 with accompanying values of A and B of 0.05 and 0.10, together with the required values of r, the number of tests necessary to assure a required reliability coefficient.

To summarize: we have described the use of sampling methods of quality control for the purpose of assuring a given reliability coefficient. Given acceptance region A and B and the ratio of T_1/T_2, it is possible to determine the number of tests, R, and the value of C such that the probability of accepting the reliability of the system, if satisfactory, will be at least 1-A and the probability of rejecting the reliability of the system, if not satisfactory, will be at least 1-B.

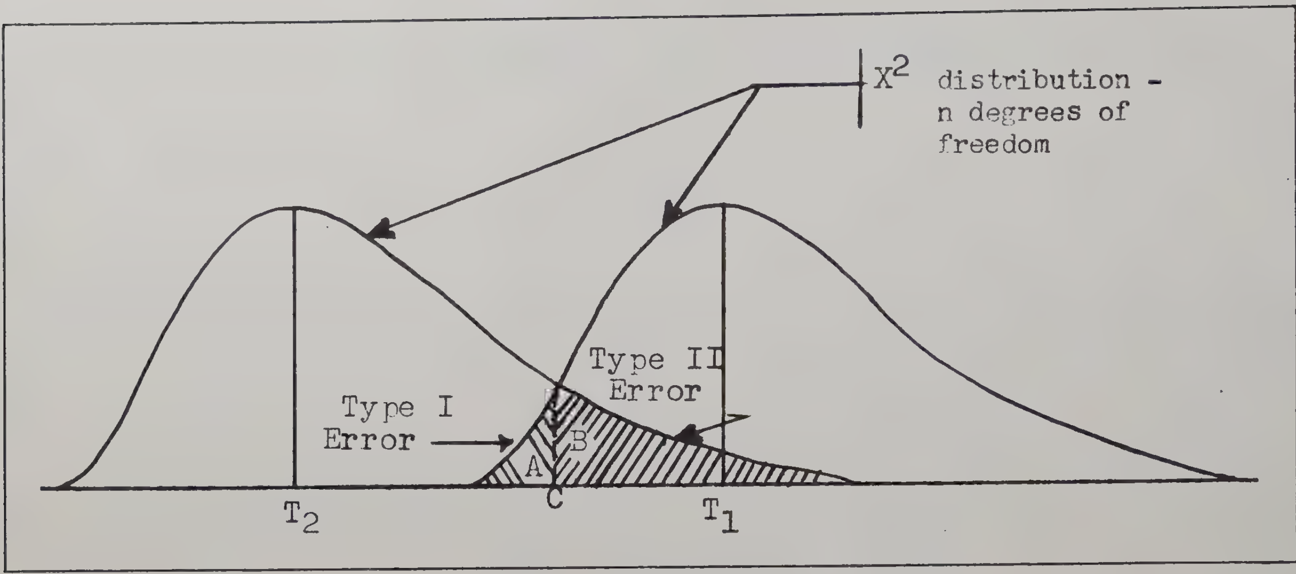

Fig. 1 - Errors of estimating T.

For example, suppose we wish to establish a reliability of 0.72 for a bombing and navigation system which will operate for a ten-hour mission. Then

$$
\begin{aligned}
R &= e^{-t/T} \\
0.72 &= e^{-10/T} \\
10/T &= 1/3 \\
T &= 30.
\end{aligned}
$$

The mean life of the system must be at least 30 hours to insure the required reliability. We must estimate the mean life by a testing procedure. The estimate of T was given previously.

If we estimate T from our sampling procedure, what confidence can we have in the estimate? In other words, what T is required to assure with a 0.95 confidence limit that the mean life will be at least 30 hours? Suppose we select as an alternative that the mean life is equal to 20 hours. This would imply a reliability of approximately 0.60. We would then test the hypothesis H_o: $T \geqslant T_1 = 30$ hours against the alternative H_1: $T < T_2 = 20$ hours. Then, the ratio $T_1/T_2 = 30/20 = 3/2$.

Consult Table I for $A = 0.05$, $B = 0.10$, $T_1/T_2 = 3/2$ for a 0.95 confidence limit and for a Type II error of 0.10. There you find $r = 55$ and $C/T_1 = 0.789$. We obtain $C = 0.789 \times 30 = 24$. This gives the desired test information, and we determine the following test: accept H_o if $\widehat{T} \geqslant 24$ after 55 test runs; accept H_1 if $\widehat{T} < 24$. This indicates that the reliability is 0.60 or less and that the probability that the reliability is as good as 0.72 is less than 0.10.

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A RELIABILITY ANALYSIS OF THE EFFECTS OF NUCLEAR RADIATION ON THE ELECTRICAL PROPERTIES OF CAPACITORS

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The mechanism of radiation damage to capacitors is quite complex, and little information is presently known regarding the nature of damage from a microscopic standpoint. For this reason, radiation effects on capacitors must be based on experimental data. Since the field of radiation effects is quite new, available data are usually on room temperature or uncontrolled temperature conditions during irradiation. Many of the conclusions drawn must then be considered as directional only.

The first atomic explosion in July, 1945, ushered in the dawn of the Atomic Age. Since then atomic-powered submarines and power stations have been in successful operation. The greatest challenge, however, is the design of a nuclear-powered aircraft. The aircraft designer is now faced with a new environment, a new parameter. Nuclear radiation has adverse effects on both electrical and mechanical properties of organic and inorganic materials. The purpose of this paper is to establish an attitude toward the radiation damage of capacitors where mixed reactor fluxes of both gamma photons and high energy neutrons are impinging upon the dielectric materials.

Figure 1 illustrates a typical set of laboratory data of y in terms of x. The independent variable x is the irradiation parameter, and the dependent variable y is the capacitance value at a specific time in the reactor. In other words N capacitors are being irradiated and each capacitor has M data points taken. Figure 2 shows this data in graphical form. A smooth curve is drawn through the data points and is labeled $y = f(x)$. The standard sample deviation is defined as:

$$s^2 = \frac{1}{N-1}\left[(y^2_{ave}) - \frac{(y_{ave})^2}{N}\right].$$

This deviation, which is a constant in this analysis, may be used to estimate a confidence band within the limits of the test data. Referring to confidence tables which may be found in most mathematics handbooks, a value l is chosen which is a function of probability and sample size. The value of l sets an upper and lower limit on the confidence band. The observer is γ per cent confident that the variability of the data lies within this envelope. Since most capacitors have tolerance limits, a maximum and minimum limit is placed on the data. In this case the capacitance exhibits a general decreasing trend so only the minimum limit is employed. As the edges of the envelope exceed this limit a pessimistic threshold (T_1) and an optimistic threshold (T_2) are established. The pessimistic threshold is the maximum amount of irradiation the capacitors will withstand at γ per cent confidence to yield a reliability of R. The optimistic threshold T_2 will not be considered since it has no intrinsic value.

In Fig. 3, as different values of reliability are chosen, the distance ls varies and intercepts the engineering limit along the abscissa. Figure 4 shows

ENERGY ABSORBED

CAPACITANCE

x/y	x_1	x_2	x_3	...	x_m
y_1	y_{11}	y_{12}	y_{13}	...	y_{1m}
y_2	y_{21}	y_{22}	y_{23}	...	y_{2m}
y_3	y_{31}	y_{32}	y_{33}	...	y_{3m}
⋮	⋮	⋮	⋮		⋮
y_n	y_{n1}	y_{n2}	y_{n3}	...	y_{nm}
	y_{1ave}	y_{2ave}	y_{3ave}	...	y_{mave}

Fig. 1

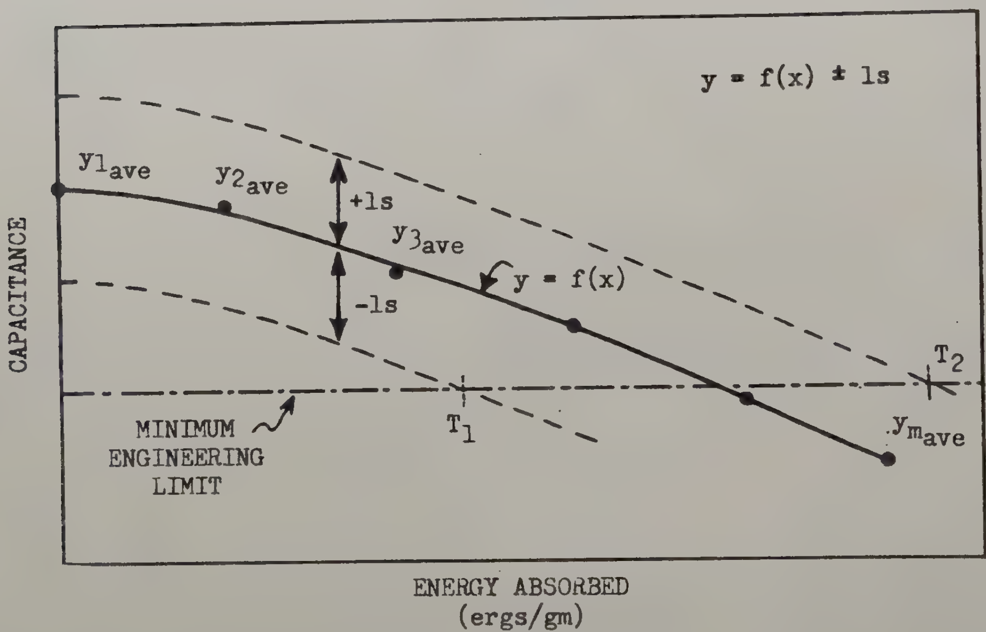
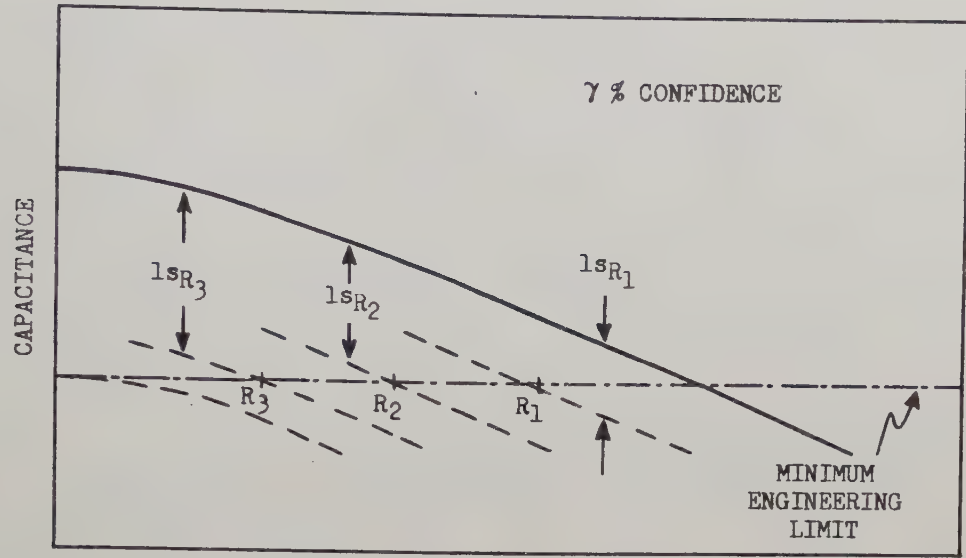


Fig. 2

these values of reliability plotted against the capacitor's functional threshold. It is interesting to note that the functional threshold curve is similar in shape to that of the raw data points. High quality capacitors have been found to have a low variability among the individual data points yielding high values of reliability during irradiation.

It should be emphasized that the analysis technique does not attempt to provide an exact appraisal of the reliability of a component; rather it provides a certain degree of confidence that the reliability of a capacitor will not be below the calculated curve. As such, the reliability shown here is not only a

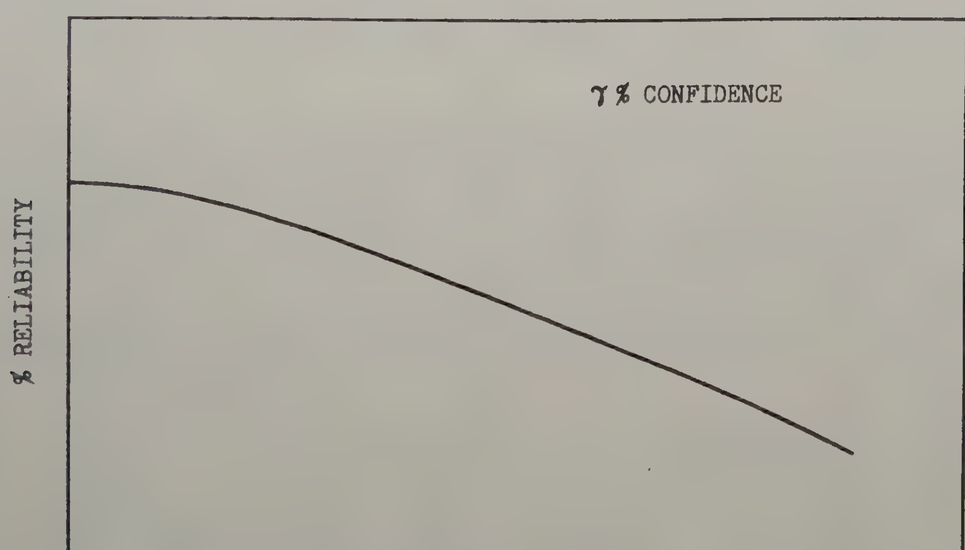


ENERGY ABSORBED IN DIELECTRIC
(ergs/gm)

Fig. 3

function of the performance of the component but also a function of the number of components tested; an increase in the number of specimens would raise the reliability curve.

A large variety of materials are employed in the construction of capacitors. Both organic and inorganic materials are used as the dielectric. The plates of capacitors are metallic and are generally radiation-resistant with respect to other materials. The integrity of capacitors is in large measure dependent upon the vulnerability of the dielectric.



FUNCTIONAL THRESHOLD
(ergs/gm)

Fig. 4

Functional threshold determination must recognize both physical and electrical degradation as caused by irradiation. For example, the electrical properties of a capacitor may remain within design specifications but fail because of structural degradation in a nuclear environment. Factors which affect dielectric materials in nuclear environments are accelerated aging due to changes in conductivity. High energy radiation produces ionization in the dielectric, flashover, lower breakdown voltages, and lower values of leakage resistance.

The material is internally heated by energy absorption from the radiation field. This energy absorption may be calculated from the following equation:

$$E_a = (\phi_\gamma \bar{C}_\gamma + \phi_n \bar{C}_n)t$$

where E_a is the energy absorbed in ergs/gm

ϕ is the radiation flux in gammas or neutrons/cm^2-sec

$\bar{C}$ is the energy absorption coefficient in $\dfrac{\text{ergs/gm}}{\text{gammas or neutrons/cm}^2\text{/sec}}$

t is the time in seconds.

The energy absorption coefficient which was developed at Convair by Brown and Fink is the subject for a paper itself, so we will define it as a conversion factor of irradiation flux to energy absorbed in a particular material.

Figure 6 illustrates the nuclear and other irradiations more commonly encountered. The neutrons which are present in the environment have velocities which range from slow to intermediate to fast. You will notice from the numbers shown here that the corresponding range covered in energy is tremendous, ranging from essentially zero kinetic energy to 10 million electron volts. The electron volt is a widely used unit of energy and is equal in magnitude to the gain in kinetic energy of an electron falling through a potential of one volt. Gamma irradiation, which is electromagnetic in nature possesses quantum energies ranging from 10^3 to 10^7 electron volts. X-radiation and visible light are forms of electromagnetic radiation familiar to all of us. From this summary it can be seen that they differ from gammas in that they have much lower energy levels.

A brief summary of some of the terms we use in discussing this environment is shown in Fig. 7. We frequently speak of the flux of nuclear radiation --

RADIATION EFFECTS IN DIELECTRICS OF CAPACITORS
Change in conductivity (rate effect)
Ionization
Lower breakdown voltages
Lower leakage resistance
Flashover
Internal heating

Fig. 5

TYPES OF RADIATION	
Neutron	
Slow	0 - 1 ev
Intermediate	1 - 10^5 ev
Fast	10^5 - 10^7 ev
Gamma	10^3 - 10^7 ev
X-Ray	10^1 - 10^5 ev
Visible Light	1 ev

Fig. 6

RADIATION UNITS	
Flux ---------	(particles or energy)/cm^2-sec
Integrated flux --	(particles or energy)/cm^2
Dose rate ---------------------------	Rem/hr
Dose ------------------------------------	Rem
Energy absorbed -------------	Rad or ergs/gm
Energy absorption rate -----------	Rad/hr or ergs/gm-hr
AEC tolerance dose rate ---------	0.3 Rem/wk

Fig. 7

either gammas or neutrons -- where we are referring either to the number of nuclear particles/cm^2-sec impinging upon a receiver or to the amount of energy/cm^2-sec impinging upon these receivers. If a flux is integrated with respect to time, the quantity referred to as the integrated flux is obtained, which simply represents the total number of particles/cm^2 or energy/cm^2 which is incident upon the receiver. From these considerations we say that our receiver, whether human or inanimate, is receiving a certain dose of irradiation.

We measure the rate at which this dose is being given in a unit known as rem/hr, and a dose received as a result of prolonged exposure we measure in rem. This dose, of course, is simply the dose rate multiplied by the time. Not all the energy which falls upon a receiver is absorbed. To measure that energy which is absorbed, another unit is used, namely the rad. By definition the rad is 100 ergs of energy absorbed per gram of material. In order to give a feeling for the size of these units, the Atomic Energy Commission has decided that personnel working in radiation fields for 40 hours/week can receive 7.5 millirem/hour with no ill effects.

Deleterious effects can be divided roughly into two categories. Some occur immediately when the component is placed in a nuclear environment and are sensitive functions of nuclear irradiation flux density; these are termed rate effects. Long term degenerative effects associated with the total irradiation are called integral effects. The damage to a given component will depend upon the modus operandi during irradiation either static or dynamic, the materials comprising the component, the intensity and length of the radiation period, and other environmental factors such as reactor temperature and humidity.

Figure 8 shows the leakage resistance of several capacitors as a function of the energy absorption rate in the dielectric. Vendor and military specifications set a minimum leakage resistance for each capacitor. When this leakage resistance exceeds this value, the aircraft designer considers it a failure. A reliability analysis was not used here since only one capacitor of each type was irradiated.

Consider a plan view of a nuclear-powered aircraft with isodose-rate lines due to the influence of the reactor for both gamma and neutron fluxes. Assume

that a capacitor in question to be placed in a particular electronics package has previous radiation curves which are available to the aircraft designer. He may predict whether or not this capacitor will exceed its specification limit due to rate effects of the flux or to integral effects over a specified flight period. From previous discussion we may conclude that the leakage resistance of the capacitor is dose-rate sensitive and the capacitance is integral-rate or dose sensitive. It is, therefore, obvious that considerable data must be available encompassing each type component to be used in a nuclear-powered aircraft so that the designer may judiciously position the various systems in the aircraft.

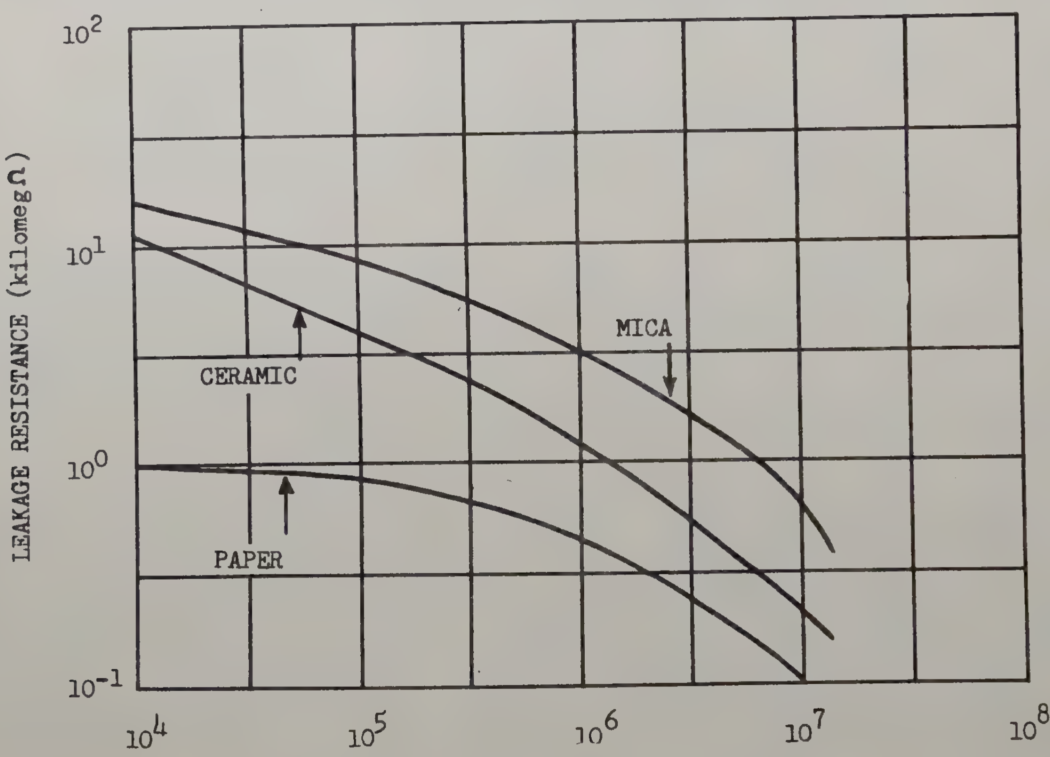


Fig. 8

The frontiers of space no longer live in the realm of dreams. Eventually aircraft guided and propelled by nuclear energy will leave the earth to venture across interplanetary distances. From the hearts of atoms the energy of the universe has been released on earth. But whether nuclear energy will eventually destroy our present world or help to create a new and better one lies not in the hearts of atoms but in the hearts of men. Human hate and ignorance or human love and knowledge are the masters. The atom will survive either.

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MULTIENVIRONMENTAL LIFE TESTING OF PARTS AND COMPONENTS IN ROCKETS AND GUIDED MISSILES BY STATISTICAL DESIGN

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Summary -- This paper suggests a method of studying the effect of environmental factors on missile parts and components for the following cases:

1. When the effects of preoperational environments are being investigated.

2. When environmental test facilities are sufficiently sophisticated to allow simultaneous application of operational environments to be made while the item is functioning.

3. When the first two conditions are satisfied and both types of information are being studied.

The investigation of the behavior pattern of the items over the environmental space is carried out by combining censored life testing with statistical experimental designs and illustrating the results by response surfaces.

THE GENERAL PROBLEM

It is well known that because of the complexity of a missile, any part or component, etc., associated with it must necessarity have an extremely high reliability number. To demonstrate or investigate the reliability of a component is very difficult. To show that the component will function for the minimum time without malfunction requires such a large sample size that it is impractical. Even if this could be done, the reliability demonstrated would be true only for the conditions and environmental combination under which the test was run. In addition, a large number of environmental combinations are possible. Higher level system interactions further complicate and vitiate the value of component reliability estimates. Finally, because of the number of components involved, each must have an extremely high reliability target.

At the part level, and even at the component level, it is not particularly realistic to attempt to demonstrate an absolute reliability number because the subsystem interactions cannot be easily generated or simulated when testing isolated components. It is more satisfactory to subject the components to a variety of environmental factors so that the best of several types of components and their reactions to the environment can be determined. Component testing may be regarded as a means of improving reliability; subsystem testing, a way of determining a realistic prediction of reliability and guiding component improvement by systematically discovering component problems and weaknesses within the higher systems and thereby establishing a natural priority of reliability apportionment. However, when it is established that a component or part is critical within a system, it then becomes necessary to discover the cause of the trouble by investigating the environmental factors and the intensities of the factors which create this situation. If it can be shown that a significant correlation exists between failure rates and certain environments, then appropriate corrective action

can be taken. It is the purpose of this paper to suggest how the correlations may be established.

LIFE TESTING AND EXPERIMENTAL DESIGN

Life Testing

Life testing, or test to failure, reduces the sample size by extracting a maximum amount of information from each sample and instead of measuring an attribute (success or failure) we are measuring a variate, time to failure. Variate sampling, being more sensitive and informative, is preferable. Life testing demonstrates how long the part or component will withstand the environmental stress compared with the length of operation required of it. However, this is normally done for one environment at a time, which is essentially the classical method of experimentation; i.e., it does not take into effect the influence of the environments which, in operational use, act and change their groupings and levels simultaneously. Therefore, in order to arrive at a realistic idea of how the different factors and their different levels of intensity influence the behavior of the items being tested, the techniques of statistical experimental design and analysis should be applied.

Because of test facility availability, expense, etc., it is not always desirable to continue each test until the unit fails; thus the slightly more complicated method of censored life testing will be discussed in this paper. The methods suggested, however, apply equally well to "simple" life testing wherein all units are tested to failure. Censored life testing is the name given to the method of testing for a given length of time and then terminating the operation, having measured the length of operation of each of those units which failed before "shutdown" and the number of units which were still functioning at time of termination.

The estimation of the life distribution is carried out as follows: Consider the estimation of the life distribution under the environmental combination

$$(A_i\ B_j\ C_k\ldots) = (ijk\ldots).$$

It is assumed that the life distribution is described by the exponential distribution and is given by

$$f(t) = \mu_{ijk\ldots} \exp\left(-\mu_{ijk\ldots} t\right) \tag{1}$$

where $\mu_{ijk\ldots}$ is the mean failure rate for environmental combination (ijk...). Test N items and, instead of allowing all items in the sample to fail, terminate the test after time T. The problem is to estimate $\mu_{ijk\ldots}$. Define the following:

T = termination time
N = total number of items in the sample
n = number of items failing before time T
t_r = time of failure of the rth unit ($r \leq n$).

It can be shown[1] (see Appendix A) that the maximum likelihood estimator of μ is given by

$$
\left.\begin{aligned}
\hat{\mu}_{ijk\ldots} &= \frac{n}{\sum\limits_{r=o}^{n} t_r + (N-n)\,T} \\
\text{and} \qquad \operatorname{var} \hat{\mu}_{ijk\ldots} &= \frac{\mu^2}{N(1-e^{-\mu T})}\,.
\end{aligned}\right\} \qquad (2)
$$

Therefore, under any environmental combination applied to N items for time T, by observing n and $\sum\limits_{r=o}^{n} t_r$ and computing $\mu_{ijk\ldots}$ from Eq. (2), the life frequency distribution for the environmental (ijk...) is completely defined.

Experimental Designs

Now suppose a censored life test be carried out for various values of i,j,k, ..., then for each combination we can estimate a life distribution in the manner described above. However, if the values of i,j,k,... are properly chosen; i.e., the correct combinations are determined by the considerations followed when establishing a statistical design, then a table of $\mu_{ijk\ldots}$ for all relevent values of i,j,k,... can be formed which will be similar to Table I and which can be used in two ways.

Firstly, a response surface can be generated, thus illustrating the behavior characteristics of the tested item over the environmental space; and secondly, after a suitable transformation[2] (see Appendix B) of the data within Table I, the techniques of analysis of variance can be utilized in order to determine significance levels and allow confidence surfaces to be plotted. The testing, therefore becomes a statistical design replicated N times with the same termination time T for each combination. In order to illustrate the foregoing statements and also prevent the notation from becoming too cumbersome, a three-factor environmental example will be given.

EXAMPLE: THREE-FACTOR ENVIRONMENTAL STUDY

A study is to be made of the effect of the preoperational environmental factors upon a component. The environments considered in this study are vibration, humidity, and temperature cycling -- factors which would normally be experienced by a weapon system transported to a tropical area and stored prior to operational use. The following environmental levels were deemed to be representative of the expected operational stress:

Vibration	a_0 a_1 a_2
Humidity	b_0 b_1
Temperature cycling	c_0 c_1

There are $3 \times 2^2 = 12$ combinations of environmental factors and levels. If the study were carried out as a simple factorial design replicated N times, then, having subjected each group of size N to one particular environmental combination

as dictated by the statistical design, all 12 groups of components would be operated for time T, and the numbers of components failing and the times at which they failed noted. For each group or environmental combination (ijk), ($i = 0,1,2$; $j = 0,1$; $k = 0,1$), we can compute

$$\mu_{ijk} = \frac{n_{ijk}}{\sum\limits_{r=o}^{n_{ijk}} t_r + (N - n_{ijk})\, T}$$

where N and T are common to each group. We can then form the array given in Table I.

TABLE I

Mean Life Estimates Under Applied Environmental Combinations

Temperature Cycle	Vibration					
	a_0		a_1		a_2	
	Humidity					
	b_0	b_1	b_0	b_1	b_0	b_1
c_0	μ_{000}	μ_{010}	μ_{100}	μ_{110}	μ_{200}	μ_{210}
c_1	μ_{001}	μ_{011}	μ_{101}	μ_{111}	μ_{201}	μ_{211}

Response Surface Example

The parameter μ_{ijk} determines the life distribution of the component under environmental combination (ijk). Therefore, from Table I we can generate environmental time-to-failure response surfaces similar to the one shown in Fig. 1. This diagram is constructed as follows: take the mean of the μ's for each level of vibration, which gives the expected value of μ_{ijk} for a_0 as

$$\mu_{o..} = \frac{\sum\limits_{j,k} \mu_{ojk}}{4} .$$

Similarly, for a_1 and a_2 the expected values are $\frac{\sum_{j,k} \mu_{1jk}}{4}$ and $\frac{\sum_{j,k} \mu_{2jk}}{4}$, which are $\mu_{1..}$ and $\mu_{2..}$, respectively.

Now these values of μ for each level of vibration are averaged values over the remainder of the environmental space. Consequently, for each life curve we have a more realistic sample of the behavior characteristics of the components. This differs from the classical or traditional method of life testing where only

the environmental factor being studied has its levels changed. The construction of a frequency environmental response surface can be carried out for each factor, graphically indicating whether there is any correlation between failure rate and environmental level.

Probability of Successful Operation

If t_o is the minimum required operating time for the component, the probability of it operating longer than t_o, the component having experienced a vibrational level of a_o is given by

$$P\ (a_o,t_o) = \int_{t_o}^{\infty} \left(\frac{\sum\limits_{jk} \mu_{ojk}}{4} \right) \exp \left[\left(- \frac{\sum\limits_{jk} \mu_{ojk}}{4} \right) t \right] dt$$

$$= \exp \left(- \frac{\sum\limits_{jk} \mu_{ojk}}{4}\ t_o \right) .$$

The surface generated and shown in Fig. 1 is a "template frequency surface" because the levels of the factors are discrete. However, if sufficient levels are studied and a regression equation between the failure rate and environmental factor can be determined, $\mu = g\ (a)$, say, where a is vibration and μ is the fail-

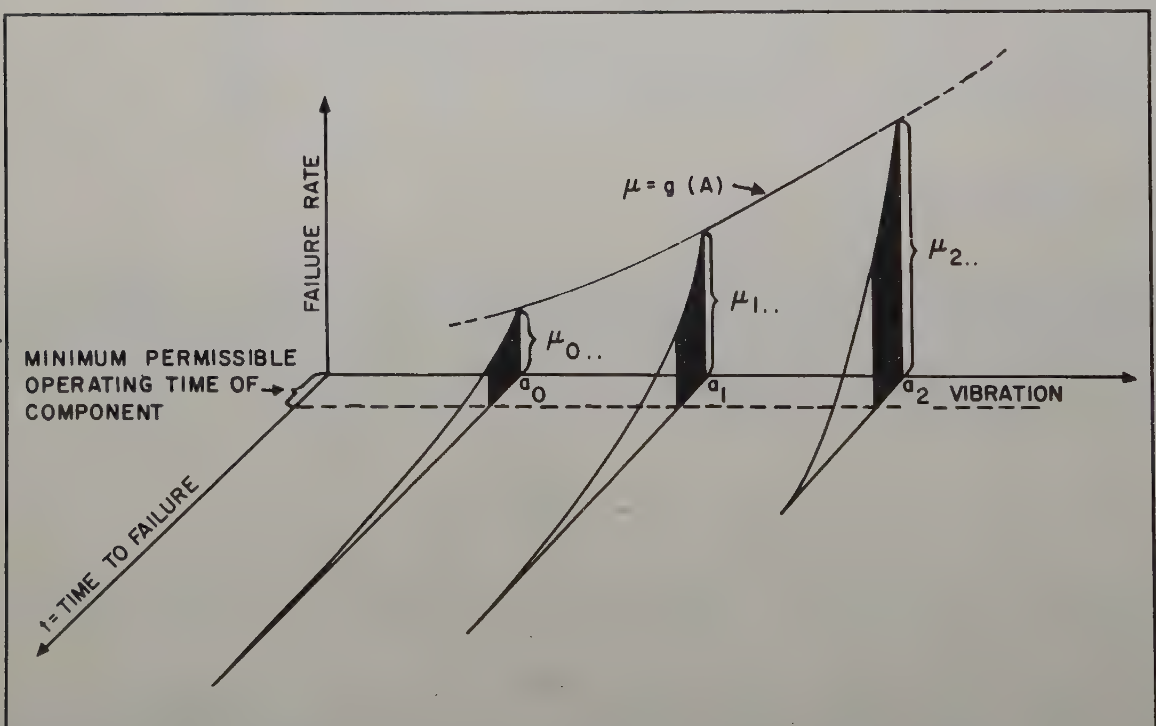

Fig. 1 - Vibration frequency response surface. (The shaded area represents the expected failure region.)

ure rate, the probability that the component will operate for a time greater than t_o, having received a vibration intensity of level a_*, is given by

$$P\ (a_*,\ t_o) = \exp \left[- g\ (a_*)\ t_o \right].$$

This, then is the probability of successful operation having received a level a_* of vibration and an average effect of all the other factors studied.

Establishing the Relative Importance of the Effect of the Environmental Factors

The above method of presentation is effective in illustrating the actual and potential trouble region of the components in the environmental space. However, much of the value of the method would be lost if it were not possible to derive a measure of variability so that significant differences between factors and their levels can be determined. It is for this reason that the environmental combinations were chosen so as to form a statistical design; i.e., an array of the type illustrated in Table I. However, the data as it stands in Table I is not in a form suitable for analysis of variance techniques, and a transformation is necessary (see Appendix B) to vitiate the relationship between μ and var μ.[2]

After the transformation has been performed the analysis of variance techniques can be applied. The significance of the effects of the different environments and their levels on the lives of the components is measurable, and we now have a quantitative basis for directing corrective action. Interactions between two or more environments which cannot be detected using the classical method of analysis will be indicated by the analysis. The example used for the statistical design is a simple factorial; however, the type of design used will depend on:

1. the number of environmental factors to be investigated

2. the number of test levels of each factor and the range of intensity of each factor

3. whether it is known or anticipated that there are important interactions between some of the factors concerned

4. the number of units available for testing and cost of testing

Much ingenuity can be exercised in arriving at the optimum design, and the very powerful tools of the analysis of variance can then be utilized to understand the life behavior characteristics of the component over the environmental space.

A Practical Consideration When Applying the Technique

The foregoing discussion is concerned with the investigation of the item's behavior and its reliability over a range of environmental conditions. However, it may not be necessary to fully investigate the behavior characteristics if during the course of the experimenting it becomes apparent that the item is reliable. For instance, we are interested in finding the areas of failure and therefore we want some of the items to fail; consequently, it is suggested that the most extreme of environmental combinations (i.e., $a_2b_1c_1$ in the example) be applied first. The reason for this is that if no units fail, or very few fail, indicating a low failure rate under the most extreme conditions, then by the theory of

peripheral testing we may extrapolate back to the nonextreme conditions and reasonably assume that the failure rate would be smaller. For instance, if we could state with a high degree of confidence that 99 per cent of the items could be expected to exceed the minimum operating time under the extreme condition, then this might be a sufficient guarantee that the item was satisfactory for all conditions without further testing. When the item or testing is expensive, it would not be prudent to spend more time or money than is necessary to assure ourselves of the item's competence.

However, if no firm decision can be deduced from the testing under extreme conditions, the "demonstration" necessarily becomes an investigation of the item's behavior characteristics, because the theory of peripheral testing can no longer be applied. Consequently, the sampling should progress from the extreme conditions in such a manner that the area of the environmental space under investigation is "self-sufficient" for a statistical analysis. Figure 2 and the subsequent explanation will clarify the foregoing statements.

Consider a two-environmental example for purposes of illustration. The most severe combination of environments is a_3b_4. If reliability cannot be inferred for the remainder of the space from the N items tested under conditions a_3b_4, extend the area of investigation to environmental combinations a_3b_3, a_2b_3, a_2b_4 and perform the statistical operations as described in the main body of the paper. This operation is extended as necessary over the environmental space until sufficient information has been derived to arrive at conclusions concerning the adequacy of the reliability, the effect of environments, and the possible need for design changes.

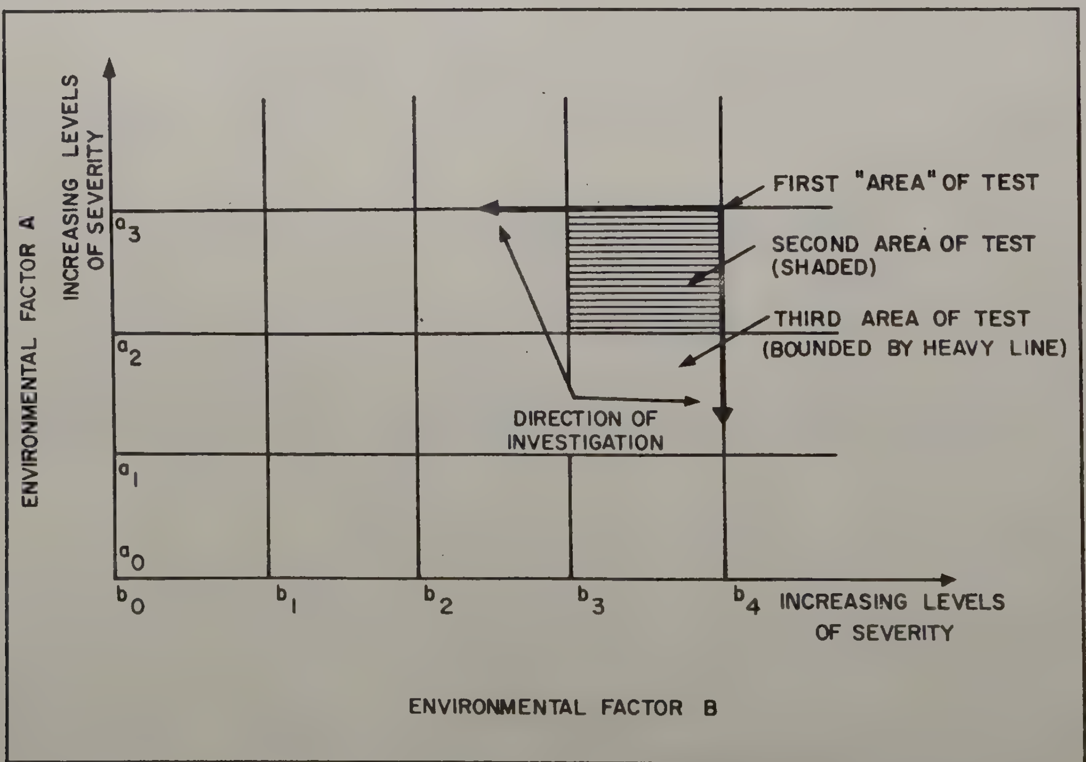

Fig. 2 - Progression of testing.

APPENDIX A

The frequency and probability functions of the life distribution are assumed to be exponential; i.e.,

$$f(t) = \mu e^{-\mu t}, \qquad 0 < t \leq T,$$

$$P = P\,(t > T) = e^{-\mu T}, \quad T < t < \infty,$$

where T is the termination time of the censored test. Taking logarithms and obtaining the first and second differentials with respect to μ,

$$\log f = \log \mu - \mu t, \quad \frac{\partial \log f}{\partial \mu} = \frac{1}{\mu} - t, \quad \frac{\partial^2 \log f}{\partial \mu^2} = \frac{-1}{\mu^2}$$

$$\log P = -\mu T, \qquad \frac{\partial \log P}{\partial \mu} = -T, \qquad \frac{\partial^2 \log P}{\partial \mu^2} = 0.$$

$$\text{The likelihood function } L = \begin{cases} {}_NC_n\, \mu^n\, e^{-\mu\left[\Sigma t + (N - n)\, T\right]}, & n > 0, \\ e^{-\mu N T}, & n = 0, \end{cases}$$

where N is the number in sample and n is the number failing before time T.

<u>The likelihood estimator of μ is given by:</u>

$$\frac{\partial L}{\partial \mu} = 0; \text{ i.e., } \frac{\partial \log L}{\partial \mu} = 0,$$

giving

$$\hat{\mu} = \frac{n}{\Sigma t + (N - n)T}.$$

<u>The likelihood estimator variance μ is given by:</u>

$$\frac{1}{\operatorname{Var} \hat{\theta}} = -n \int_{-\infty}^{\infty} \left(\frac{\partial^2 \log g(\theta)}{\partial \theta^2} \right)_{\theta = \hat{\theta}} g(\theta) d\theta$$

where $g(\theta)$ is the frequency function of the variate θ.[3] In this case, the equivalent expression is:

$$\frac{1}{\operatorname{Var} \hat{\mu}} = -N \left[\int_0^T \left(\frac{\partial^2 \log f}{\partial \mu^2} \right) f\, dt + \left(\frac{\partial^2 \log P}{\partial \mu^2} \right) P \right]$$

$$= +N\left(\int_0^T \frac{1}{\mu^2}\,\mu e^{-\mu t}\,dt + 0\right) = \frac{-N}{\mu^2}\left[e^{-\mu t}\right]_0^T$$

giving

$$\mathrm{Var}\,\hat{\mu} = \frac{\mu^2}{N(1 - e^{-\mu T})}\ .$$

APPENDIX B

In order to bring Table I into a form suitable for analysis of variance techniques, it is necessary to transform the data so that the means are independent of the variances.[2] The transformation is given by

$$I = \int \frac{(1 - e^{-\mu T})^{\frac{1}{2}}\,d\mu}{\mu}\ .$$

No exact solution was found for the indefinite integral shown, and the approximate solution is dependent on the value of μT. When μT is large the transformation is logarithmic; when μT is small the square root transformation is appropriate. However, these two transformations are useful in indicating the type of transformation required. The correct transformation will be determined by the data generated by the experiment. It is sufficient to find only an approximate transformation since the analysis of variance loses little power until the data deviate appreciably from normality.

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Index to

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PGQC-1 through PGRQC-15
1952-1958

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TABLES OF CONTENTS

PGRQC-10 June, 1957

PGRQC-11 August, 1957

PGRQC-12 November, 1957

PGRQC-13 July, 1958

PGRQC-14 September, 1958

PGRQC-15 December, 1958

INDEX TO SUBJECTS
1952-1958
(listed by year, month and page)

INDEX TO AUTHORS
1952-1958
(listed by year, month and page)